Dynamics of a Spacecraft during Extension of Flexible Appendages

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This paper deals with the attitude behavior of a spacecraft with a rotor during extension of flexible appendages. The analysis is based on the method of multiple scales. The equations of motion are formulated using a spacecraft modal coordinate scheme. The analytical expressions for the attitude behavior of the spacecraft are obtained. The extension of the appendages plays a significant role in the stability of the attitude motion of the spacecraft: In some cases, the attitude motion of the spacecraft will become unstable.

Introduction

CURRENT designs of spacecraft employ flexible deployable appendages such as antennas or solar arrays. The appendages, which are compactly stored during a launch phase, must be extended in an orbit after the initial injection. There exists much literature on the effects of the flexible appendages on the attitude motion and stability of a spacecraft (see Ref. 1, for example). The problem becomes more complex if one examines the transient behavior during the extension of the appendages. This rather brief period is critical to the success of the mission. Despite the importance of the problem many of the attitude dynamics questions on the subject seem to have remained unanswered. Several analyses on the deployment of rigid appendages from a spinning spacecraft have been described.[2,3] Deployment of flexible appendages from a spinning spacecraft have been analyzed numerically by Cherchas[4] and analytically by Honma.[5] The objective of the present analysis is to predict an unstable attitude motion of a spacecraft with a rotor during the extension period: Consider a spacecraft with a rotor and four equal flexible appendages. The appendages are mutually perpendicular in a plane normal to the spin axis of the rotor (Fig. 1). The appendages are supposed to be extended very slowly. During the course of the extension maneuver, the natural frequency of the appendages becomes close to the angular velocity of the nutational body motion (Fig. 2). In this region, the vibrations of the appendages may cause an unstable attitude motion of the spacecraft.

The method of analysis is based on the multiple scales method.[6] The results obtained are verified by those of digital simulations.

Equations of Motion

The symmetrical spacecraft chosen for the study is shown schematically in Fig. 1. A reference frame (X, Y, Z) is fixed in the main body so that the Z axis is parallel to the spin axis of the rotor. The origin is coincident with the mass center of the total system, which is assumed to be fixed at the mass center of the undeformed system. The appendages 1, 2, 3, 4 lie on the X, Y, $-X$, $-Y$ axes, respectively, in the undeformed state. The angular velocity of the spacecraft has the components $(\omega_x, \omega_y, \omega_z)$ in the (X, Y, Z) frame. The out-of-plane bending deformation of the appendage i is denoted by U_i. Since in-plane bendings of the appendages have little effect on the nutational body motion, we shall neglect them. The variables ω_x, ω_y, ω_z, and U_i are supposed to be small. The total kinetic energy T of the system is, on neglecting higher terms, given by

$$2T = I\omega_x^2 + I\omega_y^2 + I_z\omega_z^2 + J(\dot{\sigma}^2 + 2\dot{\sigma}\omega_z)$$
$$+2\left[\left\langle X\left(\frac{DU_1}{Dt} - \frac{DU_3}{Dt}\right)\right\rangle - \langle U_1 - U_3\rangle v\right]\omega_y$$
$$-2\left[\left\langle X\left(\frac{DU_2}{Dt} - \frac{DU_4}{Dt}\right)\right\rangle - \langle U_2 - U_4\rangle v\right]\omega_x \tag{1}$$

where I and I_z are the moments of inertia of the total undeformed system about the $X(Y)$ and Z axes, respectively; J is the moment of inertia of the rotor; $\dot{\sigma}$ is a relative angular velocity of the rotor; v is a velocity of the extension of the appendages; and D/Dt is the convective derivative,

$$\langle f_i \rangle \triangleq \rho \int_0^\ell f_i \mathrm{d}x \quad \text{or} \quad \rho \int_0^\ell f_i \mathrm{d}y$$

where ℓ is the length of the appendages and ρ is the mass per line element of the appendages. The elastic potential energy U, which arises from the strain energy due to deformations of the appendages, is given by

$$2U = B\left\{\int_0^\ell \left[\left(\frac{\partial^2 U_1}{\partial x^2}\right)^2 + \left(\frac{\partial^2 U_3}{\partial x^2}\right)^2\right]\mathrm{d}x + \int_0^\ell \left[\left(\frac{\partial^2 U_2}{\partial y^2}\right)^2 + \left(\frac{\partial^2 U_2}{\partial y^2}\right)^2\right]\mathrm{d}y\right\} \tag{2}$$

where B is the bending stiffness of the appendages. We assume that the dissipation function F is expressed in the form

$$F = \delta\left\{\int_0^\ell \left[\left(\frac{DU_1}{Dt}\right)^2 + \left(\frac{DU_3}{Dt}\right)^2\right]\mathrm{d}x + \int_0^\ell \left[\left(\frac{DU_2}{Dt}\right)^2 + \left(\frac{DU_4}{Dt}\right)^2\right]\mathrm{d}y\right\} \tag{3}$$

where δ is the damping factor of the appendages. Now, we expand the elastic deformation U_i as a series of the functions $E_n(\eta)$:

$$U_i = \sum_{n=1}^{\infty} T_{in}(t) E_n(\eta) \tag{4}$$

The functions $E_n(\eta)$ are defined by

$$\frac{\mathrm{d}^4 E_n(\eta)}{\mathrm{d}\eta^4} - \lambda_n E_n(\eta) = 0 \tag{5}$$

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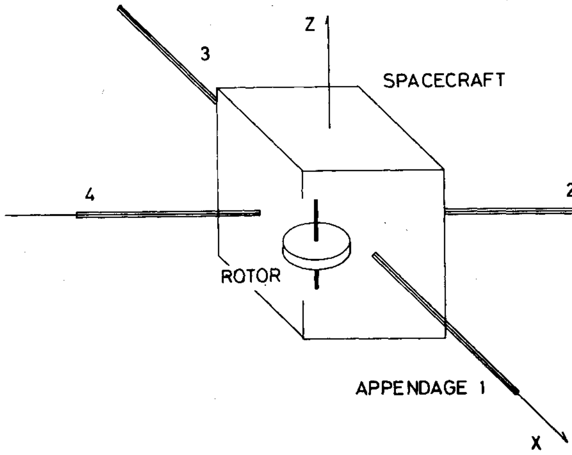

Fig. 1 Spacecraft model.

with the boundary conditions

$$E_n(\eta) = \frac{\mathrm{d}E_n(\eta)}{\mathrm{d}\eta} = 0 \qquad \eta = 0$$

$$\frac{\mathrm{d}^2 E_n(\eta)}{\mathrm{d}\eta^2} = \frac{\mathrm{d}^3 E_n(\eta)}{\mathrm{d}\eta^3} = 0 \qquad \eta = 1 \tag{6}$$

where λ_n is the eigenvalue of the normal mode $E_n(\eta)$, and $\eta = x/\ell$ or y/ℓ. In addition they are normalized such that

$$\int_0^1 E_n(\eta) E_m(\eta)\,\mathrm{d}\eta = \delta_{n,m} \tag{7}$$

where $\delta_{n,m}$ is the Kronecker delta. In what follows, attention will be paid to the case where the first mode of the vibrations of the appendages has a considerable effect on the behavior of the spacecraft. In this case, Eq. (4) can be approximated by the first mode, i.e.,

$$U_i = T_{i1} E_1(\eta) \tag{8}$$

and we shall neglect, in the following, the suffix 1.

Replacing D/Dt by $\partial/\partial t + v\partial/\partial x$ (or $\partial/\partial t + v\partial/\partial y$), and substituting Eq. (8) into Eqs. (1-3),

$$\begin{aligned} 2T = {} & I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 + J(\dot\sigma^2 + 2\omega_z\dot\sigma) + \rho\ell\dot T_i^2 \\ & + 2\rho\ell^2 a_1\omega_y(\dot T_1 - \dot T_3) - 4\rho\ell v(a_0 - a_1)\omega_y(T_1 - T_3) \\ & - 2\rho\ell^2 a_1\omega_x(\dot T_2 - \dot T_4) + 4\rho\ell v(a_0 - a_1)\omega_x(T_2 - T_4) \end{aligned} \tag{9}$$

$$2U = \sum_{i=1}^{4} (B\lambda/\ell^3) T_i^2 \tag{10}$$

$$F = \delta\rho\ell \sum_{i=1}^{4} \dot T_i^2 \tag{11}$$

where

$$a_0 = \int_0^1 E(\eta)\,\mathrm{d}\eta, \quad a_1 = \int_0^1 \eta E(\eta)\,\mathrm{d}\eta$$

and ($\dot{}$) denotes d/dt. Using Lagrange's equations for T_i and Euler's equations for ω_x, ω_y, and neglecting higher terms, we have the equations of motion

$$\dot M_x + \omega_N M_y = \rho\ell^2 a_1(\ddot T_2 - \ddot T_4) - 2\rho\ell v(a_0 - 2a_1)(\dot T_2 - \dot T_4) \tag{12}$$

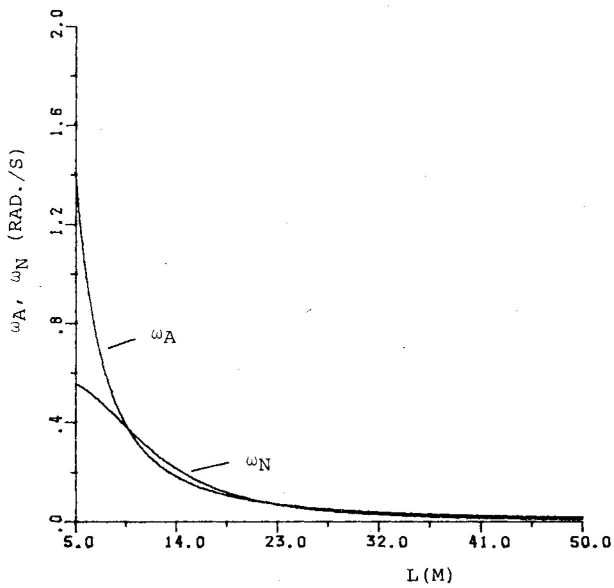

Fig. 2 Natural frequency ω_A and nutation frequency ω_N.

$$\dot M_y - \omega_N M_x = -\rho\ell^2 a_1(\ddot T_1 - \ddot T_3) + 2\rho\ell v(a_0 - 2a_1)(\dot T_1 - \dot T_3) \tag{13}$$

$$\begin{aligned} \ddot T_i + 2\left(\zeta\omega_A^2 + \frac{v}{2\ell}\right)\dot T_i + \omega_A^2 T &= \mp\frac{a_1\ell\dot M_y}{I} \mp \frac{a_0 v M_y}{I} && i = 1 \\ &= \pm a_1\ell\left(\frac{\dot I}{I^2}\right)M_y && i = 3 \end{aligned} \tag{14}$$

$$\begin{aligned} \ddot T_i + 2\left(\delta + \frac{v}{2\ell}\right)\dot T_i + \omega_A^2 T_i &= \pm\frac{a_1\ell\dot M_x}{I} \pm \frac{2a_0 v M_x}{I} && i = 2 \\ &= \mp a_1\ell\left(\frac{\dot I}{I^2}\right)M_x && i = 4 \end{aligned} \tag{15}$$

where $M_x = I\omega_x$, $M_y = I\omega_y$, $\omega_N = J\dot\sigma/I$, $\omega_A^2 = B\lambda/\rho\ell^4$, and $\delta = \zeta\omega_A$. We notice here that the change in attitude of the spacecraft is mainly due to a meridian antisymmetric mode, $T_1 - T_3$, $T_2 - T_4$, of the deflections of the appendages. Hence it suffices to take the meridian antisymmetric modes into account. This leads to

$$\dot M - j\omega_N M = -j\rho\ell^2\left[a_1\ddot T - 2\left(\frac{v}{\ell}\right)(a_0 - 2a_1)\dot T\right] \tag{16}$$

$$\ddot T + \left(2\zeta\omega_A + \frac{v}{\ell}\right)\dot T + \omega_A^2 T = 2a_1\ell\left(\frac{j\dot M}{I} - \frac{j\dot I M}{I^2}\right) + \frac{4va_0 jM}{I} \tag{17}$$

where $M = M_x + jM_y$, $T = (T_1 - T_3) + j(T_2 - T_4)$, and $j = \sqrt{-1}$. As the length of the appendages increases, the ratio of the moment of inertia of the appendages to that of the total system increases, and the natural frequency of the appendages becomes nearly equal to the nutational frequency.

In this region, the vibrations of appendages play a vital part in the attitude motion of the spacecraft. It is this region which we shall discuss below. The natural frequencies for constant ℓ are approximated by

$$\omega_{1,2} = \frac{\omega_N \pm \sqrt{\omega_N^2 + 4(1-\epsilon)\omega_A^2}}{2(1-\epsilon)} \tag{18}$$

where the upper sign is in mode 1 and the lower sign is in mode 2, and

$$\omega_3 = \frac{\omega_A^2 + \omega_A\sqrt{\omega_A^2 + 4\omega_N^2}}{2\omega_N}$$

where $\epsilon = 2\rho\ell^3 a_1^2/I$. Here, we assumed that $\zeta = 0(v)$ and neglected the terms which contain ζ in the above derivation.

In order to obtain analytical solutions of Eqs. (16) and (17), we will employ the method of multiple scales: Introduce four time scales

$$\frac{\mathrm{d}\ell}{\mathrm{d}t} = v, \quad \frac{\mathrm{d}\theta_i}{\mathrm{d}t} = j\omega_i \qquad i = 1,2,3 \tag{19}$$

In terms of the new scales, the time derivative becomes

$$\frac{\mathrm{d}}{\mathrm{d}t} = v\frac{\partial}{\partial \ell} + j\sum_{i=1}^{3} \omega_i \frac{\partial}{\partial \theta_i} \tag{20}$$

We seek asymptotic expansions for M and T of the form

$$M = \sum_{i=1}^{3}\sum_{n=0}^{\infty} v^n A_{in}(\ell)\exp\theta_i \qquad T = \sum_{i=1}^{3}\sum_{n=0}^{\infty} v^n B_{in}(\ell)\exp\theta_i \cdot \tag{21}$$

We derive the equations to determine A_{in} and B_{in} successively, substituting Eqs. (20) and (21) into Eqs. (16) and (17). Equating the coefficients of $\exp(\theta_i)$ to zero and equating like powers of v, we have for $0(v^0)$,

$$\begin{aligned} &j(\omega_i - \omega_N)A_{i0} - j\rho\ell^2 a_1\omega_i^2 B_{i0} = 0 \\ &(2a_1\ell/I)\omega_i A_{i0} - (\omega_i^2 - \omega_A^2)B_{i0} = 0 \end{aligned} \tag{22}$$

and for $0(v^1)$,

$$\begin{aligned} &j(\omega_i - \omega_N)A_{i1} - j\rho\ell^2 a_1\omega_i^2 B_{i1} = f_1 \\ &(2a_1\ell/I)\omega_i A_{i1} - (\omega_i^2 - \omega_A^2)B_{i1} = f_2 \end{aligned} \tag{23}$$

$$f_1 = -A'_{i0} + 2\rho\ell^2 a_1\omega_i B'_{i0} + \rho\ell^2 a_1\omega'_i B_{i0} - 2\rho\ell(a_0 - 2a_1)B_{i0}$$

$$f_2 = \left(\frac{2ja_1\ell}{I}\right)A'_{i0} - 2j\omega_i B'_{i0} - \omega'_i j B_{i0} - 4j\left[\frac{(\rho\ell^3 a_1/I) - a_0}{I}\right]$$

$$\times A_{i0} - 2j\left(\frac{\zeta\omega_A}{v} + \frac{1}{2\ell}\right)\omega_i B_{i0}$$

where $(\,'\,)$ denotes $\mathrm{d}/\mathrm{d}\ell$.

From Eqs. (22),

$$B_{i0} = \frac{(2a_1\ell/I)\omega_i}{\omega_i^2 - \omega_A^2}A_{i0} \tag{24}$$

Substitution of Eq. (24) into Eqs. (23) and eliminating the terms which produce secular terms in Eqs. (23), yields

$$A'_{i0} = -\delta_i^* A_{i0} \qquad \delta_i{}^* = \delta_{id}^* + \delta_{ie}^*$$

$$\delta_{id}^* = 2\omega_i^2(\omega_i - \omega_N)^2\left(\frac{\delta\omega_A}{v} + \frac{1}{2\ell}\right)\frac{1}{\epsilon\omega_i^3\omega_N + 2\omega_A^2(\omega_i - \omega_N)}$$

$$\delta_{ie}^* = \Bigg\{(\omega_i - \omega_N)4\left[\left(\frac{\rho\ell^3 a_1}{I}\right) - a_0\right]\Big/ I$$

$$+ \frac{\omega_i - \omega_N}{\rho\ell^2 a_1\omega_i^2}\left[\frac{\omega'_i\omega_A^2(\omega_i - \omega_N)}{\omega_i^2} + 4\rho\ell^2 a_1\omega_i^2(a_0 - 2a_1)/I\right.$$

$$\left. + 2C'_i\frac{\omega_A^2(\omega_i - \omega_N)}{\omega_i}\right]\Bigg\}\Big/\frac{\rho\ell^2 a_1\omega_i^3}{\epsilon\omega_i^3\omega_N + 2\omega_A^2(\omega_i - \omega_N)^2} \tag{25}$$

where $C_i = (\omega_i - \omega_N)/\rho\ell^2 a_1\omega_i^2$.

The damping coefficient δ_i for the ω_i mode is given by $v\delta_i^*$. The coefficient δ_i^* is composed of two parts; the term δ_{id}^* relating to the structural damping of the appendages and the

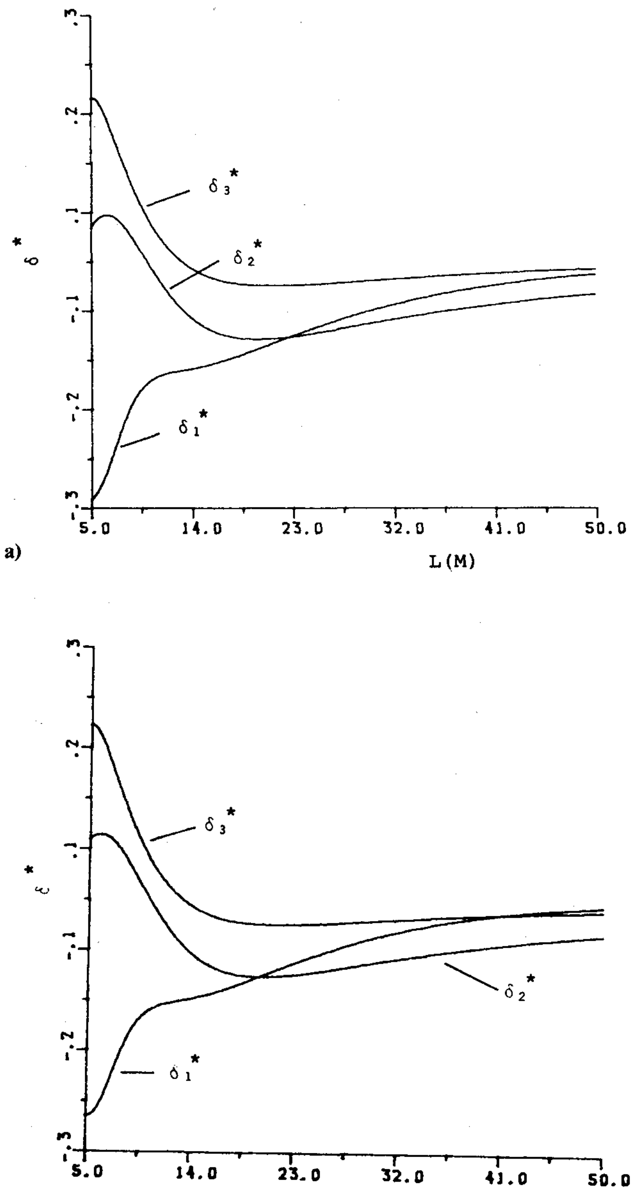

Fig. 3 Damping factor δ_i as a function of ℓ.

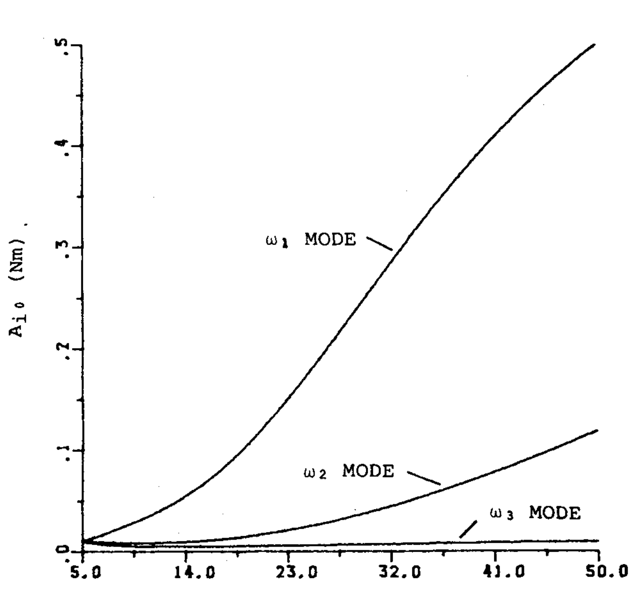

Fig. 4 Amplitude A_{i0} during extension.

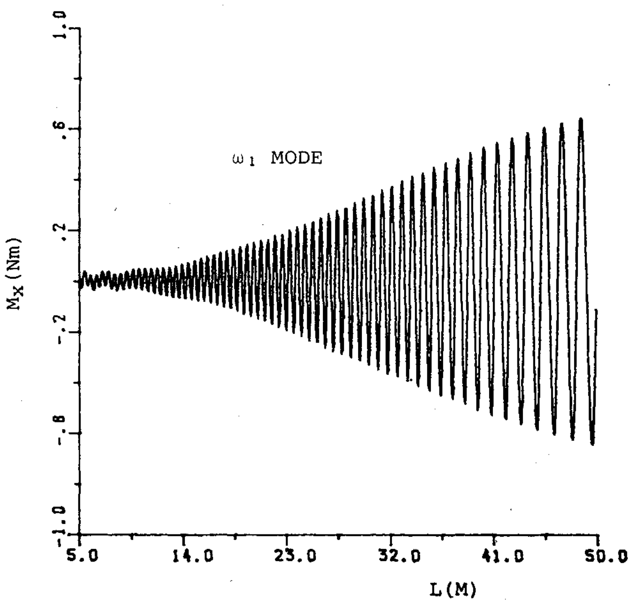

Fig. 5 Component M_x of angular momentum during extension (ω_l mode).

term δ^*_{ie} coming from the extension of the appendages. Numerical examples are shown in Figs. 3, where the system parameters are as follows:

$$\rho = 0.01\ \text{kg/m} \qquad B = 1.0\ \text{N}\cdot\text{m}$$

$$J\dot{\sigma} = 10.0\ \text{N}\cdot\text{ms}, \quad v = 0.05\ \text{m/s}, \quad I = 10.0\ \text{kg/m}^2$$

$$\zeta/v = 0.0\ \text{(case a)}, \quad 0.001\ \text{s/m (case b)}$$

It should be noted that there is a finite range where the damping coefficient δ^*_i becomes negative; the δ^*_{id} is always positive, but the δ^*_{ie} becomes negative for some period of time. The physical meaning of δ^*_i is as follows: The appendages attached to the spacecraft are extended slowly with time. When the length of the appendages are variable, the system is not closed; the system gains or loses energy by the work done on it, in addition to the dissipative forces in the appendages. For the period of time when δ^*_i is negative, the adding of energy into the system from the source of the external force (e.g., drive motor) overcomes the withdrawal of energy by the dissipation in the appendages. The amplitude of the motion of the spacecraft is, then, amplified with time. These calculations, however, do not answer the question of whether the spacecraft exhibits true instability, i.e., tumbling. The attainment of the tumbling involves nonlinear effects, and so allowance must be made for the nonlinear terms of the equations of motion, which are neglected in this analysis. For the design of this class of spacecraft, since the damping ratio ζ is usually not known, a conservative condition that $\delta^*_{ie} \geq 0$ should be used. An example numerical calculation for Eq. (25) is shown in Fig. 4, where the system parameters are the same as in Fig. 3a. The result obtained numerically from Eqs. (12-15) is shown for comparison in Fig. 5, where the envelope of M_x corresponds to A_{i0}.

The figure shows that the analytical results are in good agreement with those of the numerical simulation.

Conclusion

The attitude behavior of a spacecraft with a rotor during extension of flexible appendages has been analyzed. The damping factor of the attitude motion of the spacecraft is determined analytically and the time behaviors of the system have been also investigated. It is found that, in some cases, the attitude motion becomes unstable owing to the extension of flexible appendages. This phenomenon is considered as an unstable motion of the system characterized by the parameter which varies adiabatically with time as the result of some external action.

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